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SELECTION OF NEARBY MICROLENSING CANDIDATES FOR OBSERVATION BY THE *SPACE INTERFEROMETRY MISSION*

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ABSTRACT

I investigate the prospects for using the *Space Interferometry Mission* (*SIM*) to measure the masses of nearby stars from their gravitational deflection of light from more distant sources, as originally suggested by Paczyński and by Miralda-Escudé. I derive an analytic expression for the total observing time $T_{\rm tot}$ required to measure the masses of a fixed number of stars to a given precision. I find that $T_{\rm tot} \propto r_{\rm max}^{-2}$, where $r_{\rm max}$ is the physical depth to which candidates are searched, or $T_{\rm tot} \propto \mu_{\rm min}^2$, where $\mu_{\rm min}$ is the minimum proper motion to which candidates are searched. I show that $T_{\rm tot}$ can be reduced by a factor of 4 if source availability is extended from $V_s = 17$ to $V_s = 19$. Increasing $r_{\rm max}$ and V_s and decreasing $\mu_{\rm min}$ require a significantly more aggressive approach to finding candidates. A search for candidates can begin by making use of the Luyten proper motion catalog together with the USNO-A2.0 all-sky astrometric catalog. However, a thorough search would require an all-sky proper-motion catalog such as USNO-B or Guide Star Catalog II, which are not yet available. The preliminary observations necessary to prepare for the mission will become more difficult the longer they are delayed because the candidate pairs are typically already within 1″ and are getting closer.

Subject headings: astrometry — Galaxy: stellar content — gravitational lensing

1. INTRODUCTION

Refsdal (1964) pointed out that it should be possible to measure the masses of nearby field stars from the astrometric deviation they induce on light from more distant sources as they pass by the latter. To be practical, the two stars must pass within $\sim 1''$ of each other. Paczyński (1995, 1998) and Miralda-Escudé (1996) examined this idea in the context of the current rapid improvements in astrometric capability. They made rough estimates of the number of mass measurements that could be obtained using various ground-based and space-based facilities. Sahu et al. (1998) have found three candidates for mass measurements by imaging the neighborhoods of 500 high proper-motion stars. Here I present a systematic analysis of an idealized catalog search for candidates specifically guided by the capabilities and requirements of the *Space Interferometry Mission* (*SIM*).

The planned *SIM* launch date is 2005, although it is likely to be delayed at least a year or two. The minimum mission lifetime is 5 years. In order to carry out mass measurements, two steps must be completed prior to launch. First, since *SIM* is a pointed mission, one must identify candidate pairs of stars from a proper-motion catalog: a nearby "lens" star must be found that is likely to pass sufficiently close to a more distant "source" star to cause a large deflection of light and so permit a precise measurement of this deflection. Second, given the quality of the catalogs that will be available in the near future, it will generally not be possible to predict which of the candidates will be the best to make precise mass measurements with a modest amount of observing time. Rather, it will be necessary to perform preliminary observations of these candidates prior to the event in order to determine the impact parameter (angular separation β at the point of closest approach). Typically, the candidates are *already* closer than 1″, often much closer. Moreover, in many cases, one star is substantially brighter than the other. Hence, the preliminary observations will usually require adaptive optics or the *Hubble Space Telescope*. These requirements will grow more severe as time passes.

Because *SIM* observing time comes at a high premium, my approach is to rank candidates by the amount of observing time that is required to make a mass measurement of fixed precision. I then use this framework to characterize and evaluate various selection strategies.

The probability that it is possible to measure the mass of a given foreground star grows monotonically with its proper motion and is linear in the proper motion in most cases. Hence, a survey based on an ideal star catalog (not affected by magnitude limits or crowding) would investigate foreground stars down to some minimum proper motion $\mu_{\rm min}$. On the other hand, for stars of sufficiently low luminosity, the magnitude limits of the underlying catalog will impose an effective distance limit, $r_{\rm max}$. Thus, it is important to consider both forms of selection. In practice, the actual selection process may also be affected by crowding, but I will not consider crowding explicitly in this paper. Rather, one may think of crowding as imposing an indirect constraint on $\mu_{\rm min}$ or $r_{\rm max}$.

I derive simple expressions for the total observing time $T_{\rm tot}$ needed to make mass measurements from $\mathcal{N}$ lens-source encounters. For fixed $\mathcal{N}$, I show that $T_{\rm tot} \propto r_{\rm max}^{-2}$ for distance-limited surveys, and $T_{\rm tot} \propto \mu_{\rm min}^2$ for proper-motion–limited surveys. Hence, minimization of the observing time requires pushing $r_{\rm max}$ out as far as possible or pushing $\mu_{\rm min}$ as low as possible. The sample of candidates will then have on average smaller proper motions, meaning that they are even closer on the sky today, thus making the preliminary observations even more difficult. In addition, I show that by extending the available sources from $V_s = 17$ to $V_s = 19$, one can decrease $T_{\rm tot}$ by a factor of 4 despite the lower flux from these fainter sources. However, to determine which of these fainter sources are really usable requires a much more precise estimate of their expected impact parameters, that is, even more precise measurements of their current positions despite the larger disparity in the source/lens flux ratio. Obviously these measurements will also become more difficult with time.

To carry out a search for candidates, it would be best to begin with an all-sky proper-motion catalog with a limiting magnitude of at least $V_s = 17$ and preferably $V_s = 19$. Such catalogs are currently being prepared but have not yet been released. In the meantime, one can make a good start using the NLTT proper-motion catalog (Luyten 1979, 1980; Luyten & Hughes 1980) in combination with the USNO-A2.0 astrometric catalog (Monet 1998). I briefly describe how to carry out such a search.

SIM can potentially measure stellar masses to $\sim 1\%$ precision. This is substantially better than typical existing mass measurements as summarized, for example, by Henry & McCarthy (1993). Such highly accurate measurements for any star would be welcome to test theories of stellar structure. However, it would be particularly valuable to obtain mass measurements of low-metallicity stars, since the current set of measured stars has no members of this type. In contrast to the traditional techniques that use visual and eclipsing binaries and so select stars roughly in proportion to their volume density, microlensing tends to select high proper-motion stars. Thus, as I discuss in § 5, one can obtain mass measurements of low-metallicity stars in much higher proportion than would be expected based on their volume density.

2. REQUIRED OBSERVATION TIME FOR AN INDIVIDUAL LENS

Consider a nearby star ("the lens") of mass M and at a distance r that passes within an angle β of a more distant star ("the source") at r_s. The source light will then be gravitationally deflected by an angle $\alpha = 4GM/(\beta rc^2)$ at the point of closest approach. Consequently, the source will appear displaced by $\tilde{\alpha} \equiv \alpha(1 - r/r_s) = 4GM\pi_{\rm rel}/({\rm AU}\ \beta c^2)$ relative to the position expected in the absence of lensing. One can therefore determine the mass of the lens by measuring this displacement, provided that β and the relative parallax $\pi_{\rm rel}$ are known. Note that

$$\alpha = 80\ \mu{\rm as}\left(\frac{M}{M_\odot}\right)\left(\frac{\beta r}{100\ {\rm AU}}\right)^{-1} . \quad (1)$$

Assuming photon-limited astrometry measurements, the total amount of observing time τ required to achieve a fixed fractional error in the mass measurement then depends on three factors. First, the measurement is easier the bigger $\tilde{\alpha}$: the fractional error for fixed observing time falls as $\tilde{\alpha}^{-1}$, and so, for fixed fractional error, $\tau \propto \tilde{\alpha}^{-2}$. Second, the measurement is easier the brighter the source magnitude V_s, $\tau \propto 10^{0.4V_s}$. Third, the time required depends on the geometry of the encounter. The geometry can be described in terms of the angular coordinates (β, λ) of the source-lens separation vector at the time $(t = 0)$ of the midpoint of the mission and the angular displacement $\mu\Delta t$ of the lens relative to the source during the course of the mission. Here β is the source-lens separation at the time of closest approach ($t = t_0$), μ is the relative source-lens proper motion, Δt is the duration of the mission, and $\lambda = -\mu t_0$. I therefore write

$$\tau = T_0\left(\frac{\tilde{\alpha}}{\alpha_0}\right)^{-2} \times 10^{0.4(V_s - 17)}\gamma\left(\frac{\lambda}{\beta}, \frac{\mu\Delta t}{\beta}\right) , \quad (2)$$

where γ is a function to be described below, and where α_0 and T_0 are convenient normalization factors. For definiteness, I will take the required mass precision to be $\sigma_M/M = 1\%$ and will arbitrarily adopt $\alpha_0 = 100\ \mu$as. The normalization of γ is set to be effectively the number of equal-duration measurements that must be made. *SIM* astrometric accuracy is taken to require 1 minute to achieve 40 μas precision in one dimension at $V_s = 17$. Then

$$T_0 = \left(\frac{\sigma_M}{M}\right)^{-2}\left(\frac{40\ \mu{\rm as}}{\alpha_0}\right)^2 {\rm minutes} = 27\ {\rm hr} . \quad (3)$$

To estimate γ, I consider sets of observations over the angular interval $[\lambda_-, \lambda_+]$ where $\lambda_\pm = \lambda \pm \mu\Delta t/2$, and solve simultaneously for six source parameters: the two-dimensional angular position at the midpoint, the two-dimensional proper motion, the parallax, and $\tilde{\alpha}$. Even though very little information can be obtained about $\tilde{\alpha}$ from astrometry measurements parallel to the direction of motion, I include such measurements in order to be sensitive to other kinds of apparent source acceleration (e.g., gravitational). Without such a check, the mass measurement could not be considered reliable. I then optimize these observations for the measurement of $\tilde{\alpha}$. Generally, the optimum configuration has roughly equal total exposure times at the point of closest approach and near the beginning and end of the experiment, and has no observations at other times. I take Δt to be 5 years, but the results would be the same for any value provided that $\Delta t \gg 1$ yr. I find that γ achieves a minimum, $\gamma \sim 10$, when $\lambda_- < -2\beta$ and $\lambda_+ > +2\beta$, i.e., when the lens traverses more than two impact parameters on each side of its closest approach to the source. For example, $\gamma(0, x) = 10$ for $x \geq 4$. Some other indicative values are $\gamma(0, 2) = 19$, $\gamma(0, 1) = 99$, $\gamma(0.75, 2.5) = 24$, $\gamma(0.25, 1.5) = 39$. Thus, if the source does not move by a distance at least equal to the impact parameter on either side of the lens during the course of the observations, γ becomes very high. In my analysis below, I will incorporate the exact values of γ for each configuration. But qualitatively one can think of γ as being

$$\left[\gamma\left(\frac{\lambda}{\beta}, \frac{\mu\Delta t}{\beta}\right)\right]^{-1} \sim \gamma_*^{-1}\,\Theta\left(\lambda - \beta + \frac{\mu\Delta t}{2}\right) \times \Theta\left(-\lambda - \beta + \frac{\mu\Delta t}{2}\right) , \quad (4)$$

where Θ is a step function and $\gamma_* = 10$.

3. OBSERVING-TIME DISTRIBUTION

From the previous section, a star with $M = M_\odot$, $r = 100$ pc, $\beta = 1''$, $V_s = 17$, and a minimal γ would require about 420 hours of observation time for a 1% mass measurement. Hence, it is prudent to consider how one might find pairs of stars with the most favorable characteristics. I begin by writing down the observing-time distribution for an arbitrarily selected sample but with lenses of fixed mass M,

$$\frac{d\mathcal{N}}{dT} = \int d^3r\, d^3r_s\, dV_s\, dv_\perp\, n(\boldsymbol{r})n_s(V_s, \boldsymbol{r}_s)f(\boldsymbol{r}, v_\perp) \times S(\boldsymbol{r}, \boldsymbol{r}_s, v_\perp, V_s, \ldots) \times \delta[T - \tau(V_s, M, b, \ell, v_\perp\Delta t, r, r_s)] . \quad (5)$$

Here $n(\boldsymbol{r})$ is the number density of lenses as a function of their position, $n_s(V_s, \boldsymbol{r}_s)$ is the number density of sources as a function of their magnitude and position, $v_\perp = r\mu$ is the transverse speed of the lens relative to the observer-source line of sight, $f(\boldsymbol{r}, v_\perp)$ is the transverse speed distribution as a function of position, S is the selection function (with possibly many variables in addition to those explicitly shown), $b = r\beta$, $\ell = r\lambda$, and δ is a Dirac delta function. Equation (5) cannot be simplified without additional assumptions. As I introduce these assumptions, I will briefly outline their impact. Some of the simplifications will then be discussed in greater detail below. The reader interested primarily in results can skip directly to equation (10), where I also recapitulate the assumptions used to derive it.

I first assume that $r \ll r_s$. This is an excellent approximation for disk lenses, although it is not as good for halo lenses. It has two simplifying effects: $\tilde{\alpha} \to \alpha$, so $\tau = \tau(V_s, M, b, \ell, v_\perp \Delta t)$, and the 3-space density of sources n_s can be replaced with the projected surface density $\phi(V_s, \boldsymbol{\Omega})$, where $\boldsymbol{\Omega}$ is position on the sky. (To be more precise, ϕ is the density of sources in the neighborhood of the *lens* position $\boldsymbol{\Omega}$.) Second, I assume that the product of the selection function and number density can be written

$$n(\boldsymbol{r})S(\boldsymbol{r}, r_s, v_\perp, V_s, \dots) \to n\Theta(r_{\max} - r)\,, \tag{6}$$

where n is now assumed to be uniform, and $r_{\max}$ is the physical depth to which the lenses are searched. In fact, this is an oversimplification. A major focus of the present study is to determine what effect the selection function has on the observing-time distribution. The best way to do this is to begin with this simplified picture. With these assumptions, equation (5) can be written

$$\frac{d\mathcal{N}}{dT} = \int dv_\perp\, dV_s \int d\beta\, d\lambda\, dr\, r^2 n\Theta(r_{\max} - r)\delta(T - \tau) \times \int d\Omega\, \phi(V_s, \boldsymbol{\Omega}) f(v_\perp, \boldsymbol{r})\,. \tag{7}$$

In this form, the integration still cannot be factored because of the correlation between the speed distribution of the lenses and the distribution of sources. I therefore assume that $f(v_\perp, \boldsymbol{r}) \to f(v_\perp)$, i.e., that the speed distribution does not depend on position. This is actually a very minor assumption, provided that the speed distribution is taken to be the average over the Galactic plane, where the majority of the source stars are. Equation (7) then becomes

$$\frac{d\mathcal{N}}{dT} = n \int dv_\perp\, dV_s \int db\, d\ell\, \delta(T - \tau) f(v_\perp) \times \int dr\, \Theta(r_{\max} - r) \int d\Omega\, \phi(V_s, \boldsymbol{\Omega})\,, \tag{8}$$

where I have made use of the definitions $b = r\beta$ and $\ell = r\lambda$. The last two integrals can be evaluated directly,

$$\int dr\, \Theta(r_{\max} - r) \int d\Omega\, \phi(V_s, \boldsymbol{\Omega}) = r_{\max}\, \phi(V_s)\,, \tag{9}$$

where $\phi(V_s) \equiv \int d\Omega\, \phi(V_s, \boldsymbol{\Omega})$ is the luminosity function integrated over the entire sky. Equation (8) then becomes

$$\frac{d\mathcal{N}}{dT} = n r_{\max} \int db\, d\ell\, dV_s\, dv_\perp f(v_\perp)\phi(V_s) \times \delta[T - \tau(V_s, M, b, \ell, v_\perp \Delta t)]\,, \tag{10}$$

Equation (10) already contains an important result: the number of lenses available for measurement at fixed observing time is directly proportional to $r_{\max}$, the physical depth to which they are searched. The four assumptions used to derive equation (10) are that (1) the lenses are much closer than the sources, (2) the lenses are uniformly distributed, (3) the lens transverse-velocity distribution is independent of position, and (4) the lens selection is distance-limited.

To further evaluate the integral, I first define,

$$G(\gamma'; x) = \int dy\, \delta[\gamma' - \gamma(y, x)]\,. \tag{11}$$

Then the integral can be written

$$\frac{d\mathcal{N}}{dT} = n r_{\max} \int d\Gamma\, dV_s\, \phi(V_s)\delta[T - \tau(V_s, \Gamma)]H(\Gamma)\,, \tag{12}$$

where

$$H(\Gamma) = \int d\gamma\, db\, dv_\perp\, b f(v_\perp) G\left(\gamma; \frac{v_\perp \Delta t}{b}\right) \times \delta\left[\Gamma - \left(\frac{4GM}{\alpha_0 b c^2}\right)^{-2} \gamma\right]. \tag{13}$$

3.1. *Analytic Estimate*

Equation (12) will be evaluated explicitly in § 3.2 below. However, it is also instructive to make an analytic estimate of this equation with the help of a few approximations. First, I assume that all the sources have the same magnitude, $\phi(V_s) = N\delta(V_s - 17)$, where N is the total number of source stars. Hence,

$$\frac{d\mathcal{N}}{dT} = \frac{nNr_{\max}}{T_0} H\left(\frac{T}{T_0}\right). \tag{14}$$

Second, I use the approximation (4) to estimate G,

$$G(\gamma; x) = (x - 2)\Theta(x - 2)\delta(\gamma_* - \gamma) \quad (\gamma_* = 10)\,. \tag{15}$$

Third, I take $f(v_\perp) = \delta(v_\perp - v_*)$, where v_* is a typical transverse speed for the lens population. Then

$$H(\Gamma) = \Gamma^{-1/2} b_0 \gamma_*^{-1/2}\left[v_* \Delta t - 4b_0\left(\frac{\Gamma}{\gamma_*}\right)^{1/2}\right], \quad b_0 \equiv \frac{2GM}{\alpha_0 c^2}\,, \tag{16}$$

where I have suppressed the Θ function that limits the range of validity to $\Gamma < \gamma_*(\alpha_0 c^2 v_* \Delta t/8GM)^2$. Combining equations (14) and (16), I obtain

$$\frac{d\mathcal{N}}{dT} = \frac{2G\rho N r_{\max} v_* \Delta t}{(\gamma_* T T_0)^{1/2} \alpha_0 c^2}\,, \quad T \ll T_0 \gamma_* \left(\frac{\alpha_0 c^2 v_* \Delta t}{8GM}\right)^2, \tag{17}$$

where $\rho \equiv nM$. The limiting condition in equation (17) comes from assuming that the first term in brackets in equation (16) is much greater than the second. Equation (17) tells us that the observing-time distribution depends on the type of lens only through its mass density ρ, its typical velocity v_*, and the cutoff which scales as $(v_*/M)^2$.

A sensible observing strategy will naturally focus on the lenses that require the least observing time. I therefore consider a program that measures the masses of all lenses requiring observing times less than some maximum, $T_{\max}$. The total observing time $T_{\rm tot}$ can then be expressed as a function of the total number of stars observed, $\mathscr{N}$, and of the other parameters:

$$T_{\rm tot} = \int_0^{T_{\rm max}} dT\, T\, \frac{d\mathscr{N}}{dT}\,, \quad \mathscr{N} = \int_0^{T_{\rm max}} dT \frac{d\mathscr{N}}{dT}\,, \qquad (18)$$

$$T_{\rm tot} = \frac{1}{3}\, \mathscr{N}^3 \left(\frac{4G\rho N r_{\rm max} v_* \Delta t}{\alpha_0 c^2}\right)^{-2} \gamma_* T_0\,. \qquad (19)$$

Equation (19) is one of the major results of this paper. It states that the total observing time required to measure the masses of a fixed number of lenses scales inversely as the square of the search depth of the sample, $r_{\rm max}$. Given the premium on *SIM* time, this result implies that the search should be pushed to as large a radius as possible. I discuss the prospects for doing this in § 4.

The total time can be written out explicitly as

$$T_{\rm tot} = 230\ {\rm hr}\left(\frac{\mathscr{N}}{5}\right)^3 \left(\frac{\rho}{0.01\ M_\odot\ {\rm pc}^{-3}}\right)^{-2} \left(\frac{v_* \Delta t}{35\ {\rm AU}}\right)^{-2} \times \left(\frac{r_{\rm max}}{100\ {\rm pc}}\right)^{-2} \left(\frac{N}{10^8}\right)^{-2}\,, \qquad (20)$$

where I have assumed a mission lifetime of $\Delta t = 5$ yr and normalized the transverse speed to a typical disk value $v_* = 33\ {\rm km\ s^{-1}}$ and the density to approximately one-third of the local stellar disk density (Gould, Bahcall, & Flynn 1997). That is, I consider that one is interested in one (or perhaps several) subsets of the whole disk population. I have also assumed a total of $N = 10^8$ stars at $V_s = 17$ over the whole sky (Mihalas & Binney 1981). This estimate incorporates a maximum observing time per object,

$$T_{\rm max} = \frac{3}{\mathscr{N}}\, T_{\rm tot}\,, \qquad (21)$$

which must be well under the cutoff in equation (17) given by

$$T_{\rm cut} = 13\ {\rm hr}\left(\frac{v_* \Delta t}{35\ {\rm AU}}\right)^2 \left(\frac{M}{M_\odot}\right)^{-2}\,. \qquad (22)$$

If $T_{\rm cut} \gtrsim T_{\rm max}$, then the scaling relation (19) ($\mathscr{N} \propto T_{\rm tot}^{1/3}$) is no longer satisfied. See Figure 1 below. The cutoff is satisfied for low-mass disk stars (assuming only 5 mass measurements are desired) but becomes more difficult for higher masses.

Another important feature of equation (19) is that $T_{\rm tot} \propto 10^{0.4(V_s - 17)} T_0/N^2$. Thus, if we compare $V_s = 17$ and $V_s = 18$, the latter are 2.5 times fainter and so require 2.5 times greater $10^{0.4(V_s-17)}T_0$, the observing time for a single astrometric measurement of precision α_0. On the other hand, there are approximately 1.9 times as many stars at $V_s = 18$. (Mihalas & Binney 1981) and so $T_{\rm tot}$ is actually smaller by a factor $\sim$0.7. It should be noted, however, that the shorter observing time comes about because the impact parameter b is typically 1.9 times smaller. In § 4, I will discuss the prospects for recognizing when such close encounters will occur.

3.2. *Numerical Estimates*

To test the estimates derived in § 3.1, I continue to approximate ϕ as a δ function, but otherwise carry out the full numerical integration indicated by equations (12) and (13). I take the velocity distribution to be a two-dimensional Gaussian with (one-dimensional) dispersion typical of foreground objects in the Galactic plane: $\sigma^2 = \sigma_U^2/4 + \sigma_V^2/4 + \sigma_W^2/2$, where $\sigma_U = 34\ {\rm km\ s^{-1}}$, $\sigma_V = 28\ {\rm km\ s^{-1}}$, and $\sigma_W = 20\ {\rm km\ s^{-1}}$. This yields $\sigma = 26\ {\rm km\ s^{-1}}$, for which the mean speed is $v_* = (\pi/2)^{1/2}\sigma = 33\ {\rm km\ s^{-1}}$ (as used in § 3.1). Figure 1 shows the total number of mass measurements that *SIM* can make in observation time $T_{\rm tot}$ for the cases where the masses are $M = M_\odot$ (*bold line*) and $M = 0.1\ M_\odot$ (*solid line*). The agreement with the analytic prediction from equation (20) (*dashed line*) is excellent. Equations (21) and (22) predict that the cutoff (induced when the impact parameter β exceeds the total lens motion during half the duration of the experiment, $\mu\,\Delta t/2$) should be at $\mathscr{N} \sim 1.5(M/M_\odot)^{-1}$, that is, $\mathscr{N} \sim 1.5$ and $\mathscr{N} \sim 15$, respectively, for the two cases shown. In fact the actual values are about 2.5 times higher. Most of this difference (a factor of 2) is due to the fact that the velocity distribution is not a δ function, and the higher speed stars are more likely to be candidates and are less affected by the threshold.

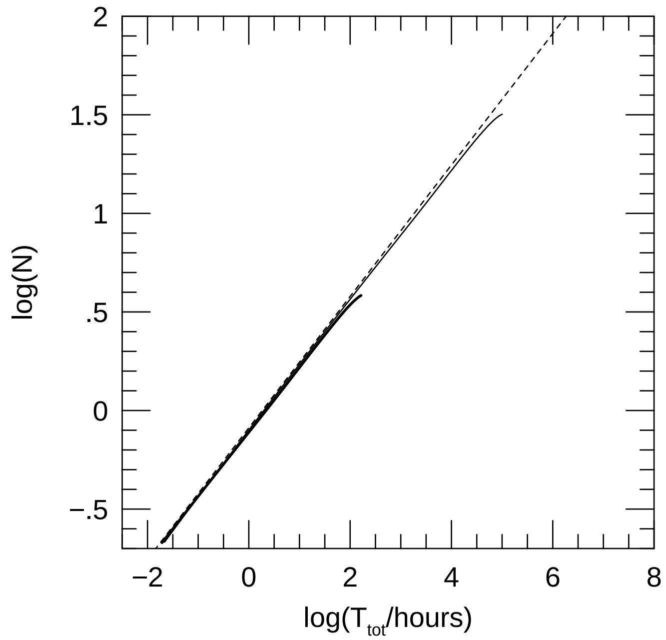

FIG. 1.—Total number $\mathscr{N}$ of mass measurements from lens-source encounters as a function of total required observing time $T_{\rm tot}$. The solid curve is for $M = 0.1\ M_\odot$, and the bold curve is for $M = M_\odot$. The dashed line is the analytic approximation (but without the cutoff) given by eq. (20).

In Figure 2 the mass is kept fixed at $M = M_\odot$, but six different cases of source-star magnitude are investigated based on the luminosity function of Mihalas & Binney (1981). The $V_s = 17$ curve (which is the same as in Fig. 1) is shown as a bold dashed line, and the others $V_s =$ 14, 15, 16, 18, 19 are shown as solid lines. The curves can be most easily distinguished by noting that the cutoff increases with magnitude. The upper bold line shows the result of combining stars in all 6 bins, while the lower bold line shows the result of combining the four bins with $V_s \le 17$. Note that each of three bins, $V_s = 17$, 18, 19, contribute about equally to $\mathscr{N}$ (for $T_{\rm tot} \lesssim 100$ hr). This is because the longer integration times required for the fainter sources are compensated by the fact that they are more numerous and hence closer on the sky on average to the lenses.

3.3. *Proper-Motion Selection*

As I discussed in the Introduction, in some regimes the selection function is best described as a cut on distance and

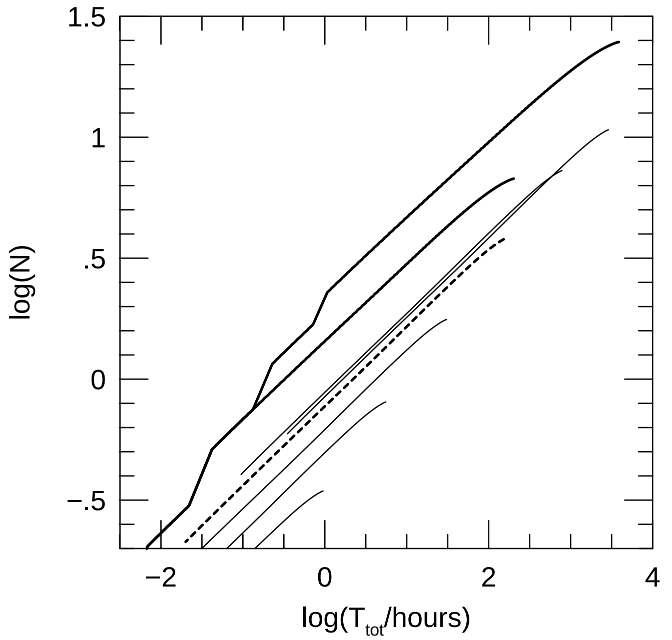

FIG. 2.—Total number $\mathcal{N}$ of mass measurements from lens-source encounters as a function of total required observing time $T_{\rm tot}$, for mass $M = M_\odot$. The $V_s = 17$ curve (same as in Fig. 1) is shown as a bold dashed line, and the others, $V_s = 14,15,16,18,19$, are shown as solid lines. The curves can be separately identified by noting that the cutoff increases with magnitude. The upper bold line shows the result of combining all of these bins ($14 \le V_s \le 19$), while the lower bold line shows the result of combining the four brightest bins ($14 \le V_s \le 17$).

in others it is best described as a cut on proper motion. So far, I have focused on selection by distance (see eq. [6]). Had I instead selected on proper motion,

$$n(\boldsymbol{r})S(\boldsymbol{r}, \boldsymbol{r}_s, v_\perp, V_s, \ldots) \rightarrow n\Theta\left(\frac{v_\perp}{r} - \mu_{\rm min}\right), \quad (23)$$

then equations (12) and (13) would be replaced by

$$\frac{d\mathcal{N}}{dT} = n \int dr\, d\Gamma\, dV_s\, \phi(V_s)\delta[T - \tau(V_s, \Gamma)]H(\Gamma; r\mu_{\rm min}), \quad (24)$$

where

$$H(\Gamma; u) = \int_u^\infty dv_\perp f(v_\perp) \int d\gamma\, db\, bG\left(\gamma; \frac{v_\perp \Delta t}{b}\right) \times \delta\left[\Gamma - \left(\frac{4GM}{\alpha_0 bc^2}\right)^{-2} \gamma\right]. \quad (25)$$

Carrying through the derivation, one obtains the analog of equation (19),

$$T_{\rm tot} = \frac{1}{3}\mathcal{N}^3 \left(\frac{4G\rho N \langle v_\perp^2 \rangle \Delta t}{\mu_{\rm min}\alpha_0 c^2}\right)^{-2} \gamma_* T_0, \quad (26)$$

where $\langle v_\perp^2 \rangle$ is the mean square transverse speed. That is, equations (19) and (26) are identical except that $r_{\rm max} v_* \rightarrow \langle v_\perp^2 \rangle/\mu_{\rm min}$. For a Gaussian, $\langle v_\perp^2 \rangle = 2\sigma^2$, so this relation can be written

$$r_{\rm max} \rightarrow \frac{4}{\pi}\frac{v_*}{\mu_{\rm min}} = 90\ {\rm pc}\ \frac{v_*}{33\ {\rm km\ s^{-1}}}\left(\frac{\mu_{\rm min}}{100\ {\rm mas\ yr^{-1}}}\right)^{-1}. \quad (27)$$

I find that with this substitution, the curves produced by equations (24) and (25) are almost identical to those produced by equations (12) and (13), except that the cutoffs are increased by a factor of 1.2. Thus, proper-motion selection and distance selection produce essentially the same results, provided they are converted using equation (27).

4. IDENTIFICATION OF PAIRS

From equation (20), the total observing time required to measure the mass of a fixed number of stars declines as $r_{\rm max}^{-2}$. From Figure 2, one sees that including the magnitude bins $V_s = 18,19$ is roughly equivalent to increasing the total number of sources N by a factor of 2, and from equation (20), the observing time is reduced as $N^{-2} \sim 0.25$. This estimate is confirmed by the offset between the two bold curves in Figure 2.

Hence, if it were possible to push out to fainter sources and more distant lenses, it would certainly be profitable to do so. I therefore now investigate the constraints governing the identification of lens-source pairs. The basic problem is that if these pairs are to be close enough for astrometric microlensing in, say, 2010, then they are *already* very close. That is, their separation $\Delta\theta_{\rm now}$ is

$$\Delta\theta_{\rm now} = 0''.6\ \frac{v_\perp}{30\ {\rm km\ s^{-1}}}\ \frac{t_0 - t_{\rm now}}{10\ {\rm yr}}\left(\frac{r}{100\ {\rm pc}}\right)^{-1}. \quad (28)$$

Thus, it would be difficult to conduct a large-scale survey for such pairs.

Fortunately, two groups are planning to release all-sky proper-motion surveys, USNO-B (D. Monet 1999, private communication), and Guide Star Catalog II (GSC II; Baruffolo, Benacchio, & Benfante 1999). Even so, the identification of pairs is not trivial, and becomes more difficult both for fainter sources and larger lens distances (or lower proper motions).

4.1. *Unblemished Survey Data*

I begin by considering the problem of candidate pair identification when the proper-motion survey data in catalogs drawn from photographic plates conform to "typical" specifications. In fact, the underlying data sets are heterogeneous, with substantially longer baselines in the north than in the south. For definiteness, I will consider proper motions derived from two epochs, one in 1955 and the other in 1990. This baseline is appropriate for the Northern Hemisphere. The anticipated proper-motion error is 4 mas yr $^{-1}$, coming from the 100 mas errors in each position measurement. This implies an error of about 120 mas in the predicted positions of the source and the lens in 2010, or 170 mas error in their relative position. (Generally, only the error in one direction—that of the impact parameter—comes into play.) Is this good enough? Let us suppose that all pairs requiring $T < T_{\rm max} = 200$ hours are to be observed. From equation (21) this corresponds to $\mathcal{N} \sim 6$. For $\gamma = 10$, equation (2) implies $\alpha \le 115\ \mu{\rm as} \times 10^{0.2(V_s - 17)}$, and so from equation (1)

$$\beta_{\rm max} = 700\ {\rm mas}\ \frac{M}{M_\odot}\left(\frac{r}{100\ {\rm pc}}\right)^{-1} 10^{0.2(17 - V_s)}. \quad (29)$$

Hence, for $M \sim M_\odot$, $\beta_{\rm max}$ is greater than 170 mas even for $r = 200$ pc *or* $V_s = 18$. Thus, while it would still be necessary to do additional astrometry in order to prepare for the observations, relatively few candidates would be rejected by this astrometry, since $\beta_{\rm max}$ is large. By contrast, for $M = 0.2$ $M_\odot$, $r = 200$ pc, and $V_s = 18$, $\beta_{\rm max} \sim 30$ mas. In this case, it would be necessary to examine about six candidates drawn from the proper-motion survey to find one suitable for a mass measurement.

In the Southern Hemisphere, the baselines are shorter, so the proper-motion errors are about twice as big. Hence, about twice as many candidates need to be examined.

4.2. *Compromised Second Epoch*

If the time of closest approach is 2010, then at the time of the second epoch of the proper-motion survey, say 1990, a typical foreground star will be separated from the line of sight to the source by of order 140 AU. This corresponds to $1''.4$ at $r = 100$ pc. As discussed in § 3.1 in the analysis of Figure 1, lens candidates tend to be moving faster than the population as a whole, so the actual typical separation will be somewhat larger. Nevertheless, these values are close to the resolution limit of the surveys and become even less favorable at greater distances. In addition, bright lens stars will entirely blot out a substantial region around them on the survey plates, preventing the detection of candidate source stars at all. For example, I find that on the Palomar Observatory Sky Survey (POSS), $V = 8$ stars (the approximate completeness limit of the *Hipparcos* catalog) tend to blot out a region with a $20''$ radius.

However, even the complete loss of the second-epoch positions of candidate sources is not crippling. The proper motion of the bright lens candidate can still be measured, and its position in 2010 predicted. This region can then be examined on the first-epoch plates for potential candidates (assuming that the lens is not bright enough to have blotted out this region even at this earlier epoch). Of course, in the intervening ~ 50 years, these candidates will have moved, but probably not by very much. For example, at high latitudes ($|b| \gtrsim 20°$), disk sources typically lie at 3 disk scale heights or about $|\csc b|$ kpc. Hence, in 50 years, they will typically move $300|\sin b|$ mas, which even at $b = 90°$ is not much more than the error in the expected position (see § 4.1). Closer to the plane, the motion will be even less. Thus, without a second epoch, more candidates will have to be examined at high latitudes (but these contain a minority of the candidates anyway) and there will be hardly any effect at low latitudes.

4.3. *What Can Be Done Now?*

The USNO-B and GSC II catalogs have not yet been released. However, it is still possible to begin the search for candidates using NLTT in combination with the USNO-A2.0 all sky astrometric catalog (Monet 1998). The conditions of such a search are fairly well described in § 4.2.

NLTT proper motions are typically accurate to about 20 mas yr^{-1} (Reid 1990). The positions are accurate only to a few arcseconds. By identifying the NLTT stars in the USNO-A2.0 survey, one could fix the ~ 1955 positions to ~ 100 mas, and hence the 2010 positions to $\sim 1''$. One could then search the USNO-A2.0 catalog for background stars whose ~ 1955 positions lay along the 2010 path of the NLTT star. As discussed in § 4.2, these stars could be expected to move about $300|\sin b|$ mas, which is generally small compared to the uncertainty in the position of the foreground star. Hence, 2010 source-lens separations could be predicted to $\sim 1''$. For pairs that were sufficiently close, the separation could be measured on the POSS II plates to refine the prediction. Additional ground-based observations could then be made of those pairs surviving this test.

After completion of the present paper, Salim & Gould (2000) undertook such a search, which turns out to be substantially more complicated than envisaged in this section. Nevertheless, they identified about 200 candidates, of which 35 have *Hipparcos* astrometry and so are very solid, and the remainder have NLTT proper motions with USNO-A2.0 positions and so require additional observations for verification.

5. STELLAR HALO LENSES

Halo stars are about 500 times less common than disk stars (Gould, Flynn, & Bahcall 1998), i.e., $\rho \sim 6 \times 10^{-5}$ $M_\odot$ pc^{-3}, and they are typically moving about 8 times faster. Let us suppose that they could be spotted to $r_{\rm max} = 1$ kpc (see below), and let us take $N = 4 \times 10^8$ in accord with the discussion of Figure 2. Then, from equation (20), it would be possible to obtain the masses of five halo stars with about 60 hours of observation.

At $r_{\rm max} = 1$ kpc, it is still appropriate to approximate the density of the stellar halo as uniform. However, it is no longer appropriate to treat the sources as being infinitely far away. As mentioned in § 4.2, at $b = 90°$, typical disk sources are at 1 kpc. However, the disk sources are farther away at lower latitudes where there are the greatest fraction of sources in any case. Hence, given the level of approximation of the present study, I will ignore this modest correction.

Of course, the radius within 1 kpc contains ~ 300 times more stars than the radius within 100 pc, so identifying a relatively complete sample of halo stars seems like a formidable task at first sight. In fact, for stars of fixed color (and approximated as blackbodies, or at least as deviating from blackbodies in similar ways), we have

$$t_{\rm cross} = k_{(B-R)0}\, 10^{-0.2R_0} \mu^{-1} , \qquad (30)$$

where $k_{(B-R)0}$ is a constant that depends on the dereddened color, R_0 is the dereddened magnitude, and $t_{\rm cross}$ is the time it takes the star to cross its own radius. For halo stars, $t_{\rm cross} \sim 10^3$ s, which is significantly different from the value for other common classes, 2×10^2 s for disk white dwarfs, 5×10^3 s for thick-disk stars, and 2×10^4 s for main-sequence stars. Thus it should not be difficult to find halo star candidates in a proper-motion catalog with colors. The sample will be somewhat contaminated with fast-moving thick-disk stars, but these are also of considerable interest because of their low metallicity.

A more significant problem is that if the survey is limited to $V \sim 20$, then at 1 kpc the bottom of the luminosity function $M_V > 10$ is not detectable. These fainter stars contain about half the spheroid mass, implying that the above estimate of the observation time required to measure 5 masses should be multiplied by 4 to about 250 hr. Of course, if one were willing to settle for 3% measurements in place of 1%, this estimate would come down by an order of magnitude.

6. CONCLUSIONS

The main results of this study are as follows:

1. The distribution of lenses that are available for astrometric mass measurements scales directly as the distance limit $r_{\rm max}$ (or inversely as the proper-motion limit

$\mu_{\rm min}$) of the survey for lens candidates. That is, if one survey probes to twice as great a distance as another, it will have twice as many candidates, but the relative distribution of candidates according to mass, required observation time, metallicity, or any other property (except distance) will be on average the same. This conclusion is derived with the aid of a few reasonable simplifying assumptions, which are summarized below equation (10).

2. The total amount of observation time required to measure $\mathcal{N}$ masses scales as $T_{\rm tot} \propto \mathcal{N}^3(v_* \rho r_{\rm max})^{-2}$ or $T_{\rm tot} \propto \mathcal{N}^3(v_* \rho)^{-2}\mu_{\rm min}^2$, where ρ is the mass density of lenses and v_* is their characteristic speed. Hence, doubling the search radius (or halving the proper-motion limit) can reduce the required observation time by a factor of 4. For equal amounts of observation time applied to stellar-halo and disk lenses, one can measure masses for a factor $\sim [(v_* \rho)_{\rm halo}/(v_* \rho)_{\rm disk}]^{2/3} \sim 6\%$ fewer halo stars. While this is a relatively small number, it is much larger than the relative halo/disk density ($\sim 1/500$) that fundamentally limits mass measurements by other techniques. In addition, as discussed in § 5, it should be possible to find halo-star candidates over a substantially larger volume compared to the disk stars.

3. Existing astrometric and proper-motion catalogs are adequate for an initial search for lens candidates. Salim & Gould (2000) have already undertaken such a search. With the arrival of new, much deeper proper-motion catalogs, it should be possible to increase the number of viable candidates by a factor of a few. However, the ratio of spurious to viable candidates will also rise by a factor of several, necessitating a substantial increase in the additional observations required to sort the wheat from the chaff.

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